

## Statistical formulation of cremated remains quantification

Let a site contain an unknown total mass of cremated remains  $M_{total}$ . Only a sample fraction  $f_s$  of the site can be recovered due to ethical and practical constraints. Let  $m_s$  be the measured mass from the sample. Then:

$$m_s = f_s \cdot M_{total} + \varepsilon_s$$

where  $\varepsilon_s$  represents measurement error from incomplete separation of cremains, soil admixture, and heterogeneity of deposition.

The estimate of total cremains is:

$$\hat{M}_{total} = \frac{m_s}{f_s}$$

with associated variance:

$$\sigma_{\hat{M}}^2 = \frac{\sigma_s^2}{f_s^2} + \sigma_p^2$$

where:

- $\sigma_s^2$  = sampling variance due to heterogeneity of cremains distribution
- $\sigma_p^2$  = propagated uncertainty from methodological assumptions (e.g., density of cremains, incomplete separation)

If spatial distribution is non-uniform, define  $n$  subsamples  $\{m_{s,i}\}$  over site partitions  $\{f_{s,i}\}$ . Then:

$$\hat{M}_{total} = \sum_{i=1}^n \frac{m_{s,i}}{f_{s,i}}, \sigma_{\hat{M}}^2 = \sum_{i=1}^n \frac{\sigma_{s,i}^2}{f_{s,i}^2} + \sigma_p^2$$

Observations:

1.  $\hat{M}_{total}$  is **highly sensitive** to  $f_s$  and to spatial heterogeneity.
2. If  $f_s \ll 1$  or distribution is diffuse,  $\sigma_{\hat{M}}$  can approach or exceed  $\hat{M}_{total}$  itself.
3. Ethical constraints limit  $f_s$ , so achievable precision is inherently bounded.
4. Archaeological evidence confirms presence/location of cremains but **cannot reliably reduce  $\sigma_{\hat{M}}$  below bounds imposed by partial sampling.**

Conclusion: numerical quantification of cremated remains is fundamentally **uncertain**, with error dominated by sampling fraction and heterogeneity. Point estimates without explicit confidence intervals are not statistically robust.